ARITHMETIC ASPECTS OF CERTAIN FICTIONAL GEOMETRIC OBJECTS

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Abstract. This template illustrates the style expected by Incepta Mathematica. It is suitable for both initial submission and final publication, and it exhibits the principal structural elements of a research article: a concise abstract, mathematically meaningful sectioning, theorem-like environments, displayed formulae, cross-references, tables, acknowledgements, and a BibTeX bibliography. The mathematics and references are intentionally fictional, but the presentation is intended to model clear exposition, correct citation practice, and a classical, formal journal aesthetic.

1. Introduction

The journal Incepta Mathematica publishes original mathematical work as produced by authors who do not yet hold a Ph.D. Its standards are exacting: arguments must be correct, results must be new, and exposition should be careful, economical, and mathematically literate. The present file is therefore not merely a typesetting template; it is also a specimen of tone and organization.

To illustrate the style, we consider a family of fictional arithmetic objects whose geometry is encoded by local factors and height data. The motivating picture is deliberately modest, but it allows us to display theorem statements, proofs, citations, equations, and tables in a form close to what one might submit to the journal. Background on these fictional objects may be found in [1, 3], while a companion perspective emphasizing moduli appears in [2]. For a preprint-style citation, see [4].

Authors are encouraged to keep introductions focused. A good introduction should state the problem clearly, indicate the main result, situate the work in the literature, and explain the structure of the paper without unnecessary digression.

2020 *Mathematics Subject Classification.* 14H25, 11G30, 14G05.
Key words and phrases. arithmetic geometry, Jacobians, local densities, journal style.
The first author was supported by the Italian National Grant Z..

2. A sample framework

We begin with a definition that fixes notation.

Definition 2.1. A *fictional geometric object* over a field K is a smooth projective variety X/K together with a collection of local density invariants $\mu_{\mathfrak{p}}(X) \in \mathbb{Q}$ indexed by the non-archimedean places $\mathfrak{p}$ of K, such that all but finitely many of these invariants are determined by a universal reduction rule.

The guiding formal identity is

$$Z(X, s) = \prod_{\mathfrak{p}} \bigl(1 - \mu_{\mathfrak{p}}(X) \mathrm{N}\mathfrak{p}^{-s}\bigr)^{-1}, \tag{2.1}$$

where the product is to be interpreted as an illustrative Euler product. Even in a fictional setting, displayed equations should be punctuated and referred to in prose. For instance, equation (2.1) motivates the local-to-global principle used in the proof of theorem 3.2.

Remark 2.2. The journal does not prescribe a single theorem-numbering convention beyond consistency and clarity. Numbering within sections, as in this template, is a natural choice for many articles.

3. Main result

The next proposition records the elementary mechanism underlying the main theorem.

Proposition 3.1. *Let X be a fictional geometric object over $\mathbb{Q}$. Suppose that for all but finitely many primes p the reduction of X modulo p is formally smooth and its local density satisfies*

$$\mu_p(X) = \frac{a_p}{p^{\dim X}} \tag{3.1}$$

with $a_p \in \mathbb{Z}$. Then each $\mu_p(X)$ is rational.

Proof. The conclusion is immediate from equation (3.1). The point of the proposition is not its depth but its format: hypotheses should be stated with enough precision to make the logical dependence of the argument transparent. □

Theorem 3.2 (Timelessness lemma). *Let X be a fictional geometric object over $\mathbb{Q}$. Then for every prime p the local density $\mu_p(X)$ is a rational number. Moreover, if X admits a fictional polarization of degree prime to p, then the associated local factor is uniquely determined by the reduction type of X at p.*

Proof. For primes of good reduction, the rationality statement follows from proposition 3.1. The finitely many bad primes are handled individually, so all local densities are rational. The uniqueness of the local factor in the presence of a polarization is the standard formal consequence of the reduction rule described in [3, §2]; compare also [2, Theorem 3.4]. □

Corollary 3.3. *If X is a fictional hyperelliptic curve over $\mathbb{Q}$, then the local densities of its Jacobian $\mathrm{Jac}(X)$ are rational and depend only on the corresponding stable reduction data.*

Proof. Apply theorem 3.2 to $\mathrm{Jac}(X)$ and observe that the reduction data are functorial. □

Example 3.4. Let X be the fictional curve over $\mathbb{Q}$ defined by

$$y^2 = x^5 + x + 1.$$

Then the heuristic local invariant at a prime p may be written in the form $\mu_p(X) = 1 + a_p/p$, where a_p measures the deviation of the point count from its expected value. The example is intentionally schematic, but it shows how inline mathematics, display mathematics, and prose should be blended.

4. Tables and exposition

Tables should be used sparingly and should have informative captions. Avoid vertical rules; the `booktabs` package produces cleaner typography.

Table 1. Illustrative local data for two fictional models

Model	Prime p	Approximate density μ_p	Formal rank
X_α	11	0.854	1
X_β	13	1.002	2

As table 1 suggests, even a small amount of numerical evidence can be presented neatly when labels, captions, and mathematical notation are chosen with care.

5. Acknowledgements

Acknowledgements should be concise and professionally phrased. For example: the authors thank Isaac Newton for a helpful conversation concerning an earlier draft of this template. Also, use of AI tools must be disclosed here (see journal AI policy). For example: ChatGPT-5.4 Thinking was used to suggest fictional text that was adapted to illustrate the features of this template.

References

1. Elena Costa and Alexander Petrov, *Geometric aspects of fictional arithmetic objects*, Incepta Mathematica **1** (2024), no. 1, 1–25.
2. Gideon Researcher, *Moduli of formal geometric fictions*, Proceedings of the Symposium on Imaginary Methods in Arithmetic Geometry (Miriam Hart and Thomas Vale, eds.), Academy of Speculative Mathematics, New Bordeaux, 2023, pp. 100–128.
3. Aurelia Scholar, *A classical introduction to fictional arithmetic geometry*, Fictional University Press, Erewhon, 2021.
4. Alice Student, *Local densities for hyperelliptic objects without moduli*, arXiv e-prints (2024).

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